
THE DOUBLE FRACTAL STRUCTURE OF VENICE

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Andrew Crompton

School of Environment and Development, The University of Manchester

Frank Brown

School of Environment and Development, The University of Manchester

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Andrew Crompton

*School of Environment and Development, The University of Manchester, Oxford Road, Manchester, M13 9PL, UK
a.crompton@manchester.ac.uk*

Frank Brown

*School of Environment and Development, The University of Manchester, Oxford Road, Manchester, M13 9PL, UK
f.brown@manchester.ac.uk*

Abstract

This space syntax study illuminates the historical development of Venice and reveals that its boundary between water and land lies at the junction of two fractals. This result is based on the analysis of newly drawn figure-ground and axial maps of the Venice that proved to be very suitable for a space syntax study being flat with a well-defined boundary to eliminate edge effects. The historical development of the city from the dense core to the looser outer parts is traced in the maps that also show evidence of filled in canals and traces of French axial planning from the time of Napoleonic conquest when alien concepts of urban planning were imposed. Separate axial maps for the street and canals show significant differences in scale and depth, which explains the different spatial experiences of Venice seen on foot and by boat. A statistical test showed that distributions of line lengths for both canals and streets were hyperbolic indicating that Venice is fractal. The distribution of axial line lengths was found to follow the power law model for cities described by Carvalho & Penn, 2004, but with different exponents for streets and canals. The streets belong to the pattern seen in older settlements where local planning is influential whereas the canals are more like modern cities where global planning is dominant and open spaces cross the entire city. These facts are combined with recent research on cognitive distance perception to explain why Venice appears so large to the pedestrian even though the parts visited by tourists cover roughly the same area as Central Park in New York, on the face of it a much smaller place. Venice may thereby be interpreted as the intersection of two fractal circulation systems, an open one for commerce and a more closed one for living. This is a very appropriate form for a city that is a byword for civilised living, which at the same time managed to handle most of the Levant trade for five hundred years.

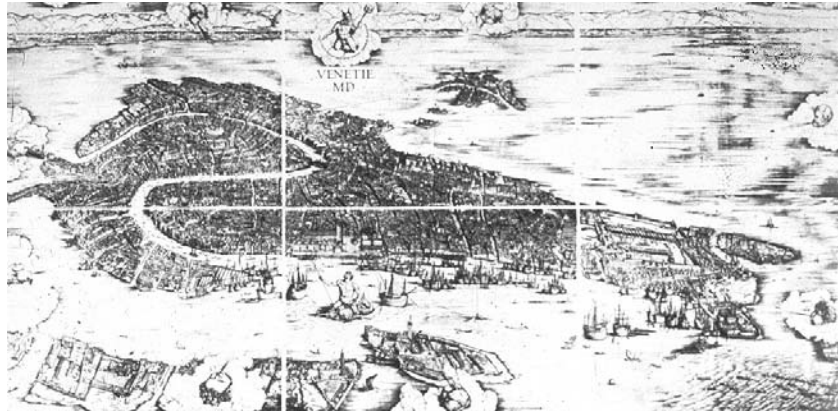
Introduction

Venice has grown from a settlement on a cluster of islands in a lagoon that expanded by landfill until only thin canals were left between them, except for the Grand Canal, which is the trace of a bend in a river. Jacopo de Barbari's well-known bird's-eye map of 1500 depicted the city as a unified built environment that had by then overwhelmed its original foundations in marsh and mud. His map, (figure 1), was constructed from ground-based observations that were fitted together using computations involving the heights of towers and their distances

from each other. The difficulties of this process accounts both for the slight disproportion in scale between horizontal and vertical and for the changes in angle between individual map pieces, (Schulz, 1978).

Figure 1:

Jacopo de Barbari's map of Venice, 1500

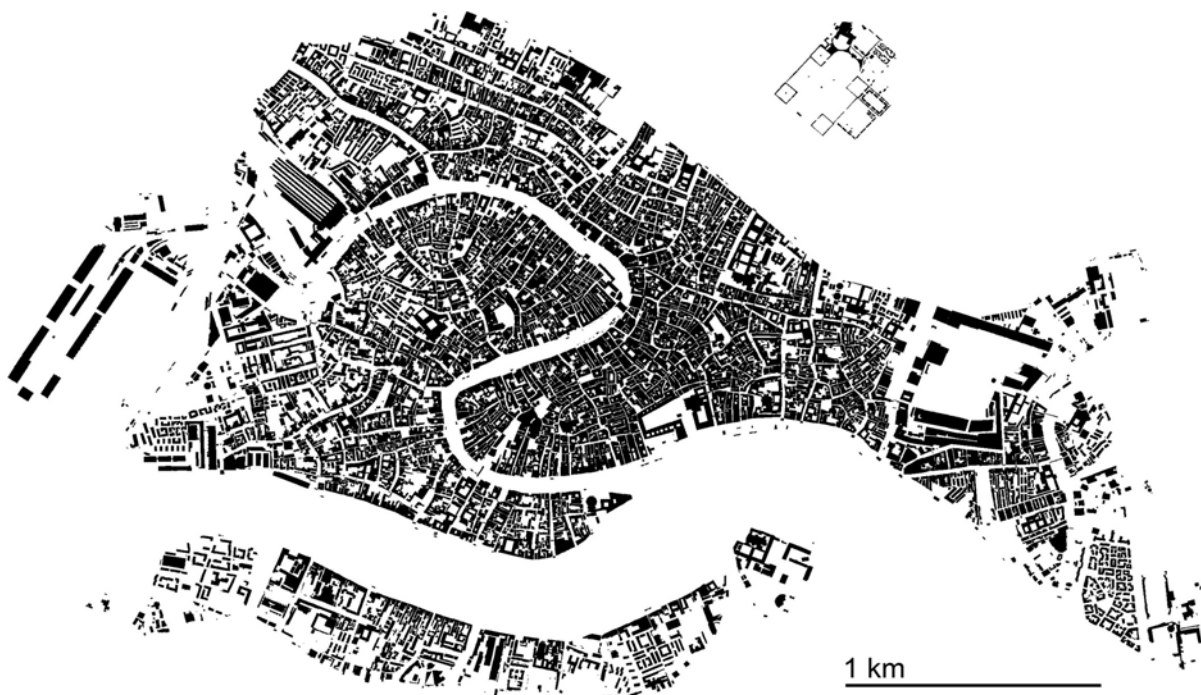


In a like manner axial maps are constructed by joining together ground level measurements to show relationships that would otherwise be as imperceptible as aerial views were before the aeroplane. They encode the structure of open spaces in a city and show how they are connected. The aim of our study was to use space syntax to look at this unusual city in a fresh way. In doing this we have discovered some unexpected regularities in its structure that relate to the idea of a fractal city as developed by Batty and Longley, (1994) and more particularly to work of Carvalho and Penn (2004) on axial maps and scaling in urban space.

It is perhaps not surprising that Venice turns out to be fractal because at one level at least it can be seen to be composed of parts resembling the whole. The regions into which the canals divide the city are relatively separate and connected to their neighbours by a few bridges so that each one is like a town in its own right surrounded by water, each in fact is like a miniature Venice. Indeed one of them, the original Ghetto, was notoriously used for the confinement of Jews because it could be sealed at night with gates on just three bridges. This is why the key to not getting lost in Venice is to know where the canals can be crossed, many passages end on the water's edge and it is easy to get lost in one of the many deep dead ends.

Figure 2:

Grain plan of Venice



Since its bridges all have steps Venice is a city that manages without the wheel and its scale, on land at least, is that of a pedestrian. Streets are generally between 2.0 and 4.5 m wide and it is common to have to queue at bottlenecks in the high season in traffic jams of people. The price of living on an island is that there is hardly any open space left, the grain plan in figure 2 shows how very dense Venice has become. It achieves maximum density in its old commercial heart close to the Ponte di Rialto that was for many years the only place where the Grand Canal could be crossed on foot, even today there are only three bridges that go over it. The largest public open spaces are a few irregularly shaped Campi, [i.e. originally fields], such as the Campo San. Polo. These and other smaller squares were important for the collection of rainwater. Only one open space was dignified with the name 'Piazza', and that is the famous Piazza San. Marco, the first open space in the city to be paved.

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Method

Venice turned out to be a very suitable candidate for a space syntax study because it is flat with a definite edge to eliminate edge effects and few tall structures to make disruptive visual links between otherwise separate spaces. Our work was based on the AutoCAD model of Venice kindly provided by Marisa Scarso of the Università di Venezia. This is a very detailed survey of the city that includes objects such as trees, wells and walls. It stops at the public-private boundary and does not include the interiors of buildings. We divided the model into layers enabling separate plans for buildings, pathways and canals to be drawn, the grain plan so made is shown above. The model shows internal courtyards but as it was not always possible to know how they are reached they were not included in our axial map. The same applied to the gardens: Venice is reputed to have three hundred and thirty three of them, usually small, walled and private. Where the model showed openings in garden walls the axial map got inside, but most of them were not reached. Had we known more about private spaces our axial map could have been taken to another level of detail, as it was it had 5274 lines, 4864 of them on land and 410 over water.

Our concern was only with the historic city, that is, with the parts that tourists visit. As may be seen by comparing the grain plan with the axial map, the Giudecca, the long island to the south was not included in our axial survey, nor was the docks and the car park to the west or the parts east of the Arsenale basin. These contain modern developments with a different scale and grain to the old parts of the city. The axial map was drawn using site notes alongside a selection of maps of Venice and aerial photographs from the book by Guerra & Scarso, (1999). Since Venice is flat we did not have to consider whether or not axes were interrupted by rising ground, but we found it necessary to assume that small bridges, which usually rise about a metre or so to cross canals did not obscure vistas, something we believe to be generally true but was impossible to check in every case. However since axes rarely cross canals we do not consider this significant. Axes were always terminated at the water edge. On water things were simpler the only difficulty being a choice about how to handle axial lines along canals that left the city and headed into open water. Traffic in the lagoon is confined to lanes marked by poles and buoys; should these be taken into account or not? Separate maps were drawn to cover both cases, the differences between them were significant but do not affect the conclusion of this study¹.

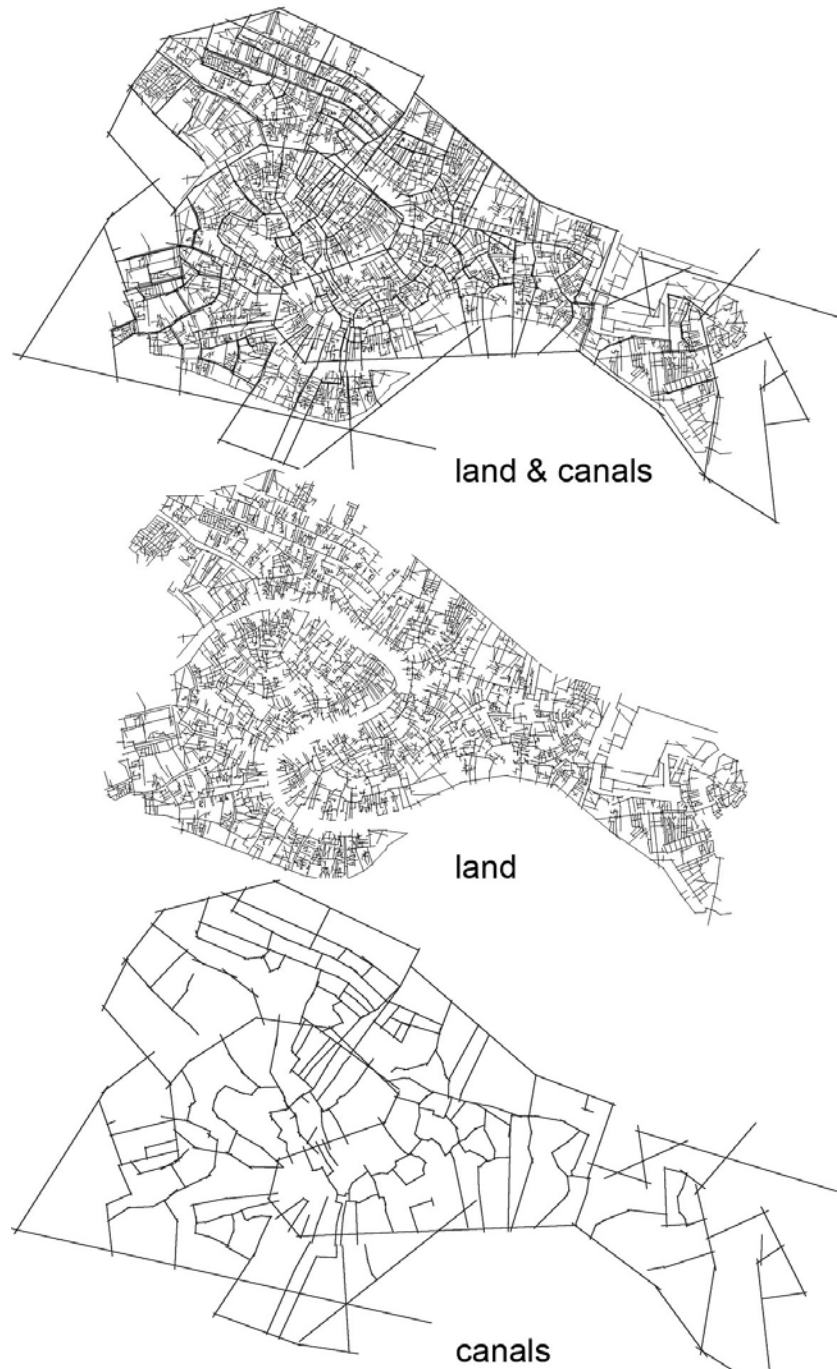
Although there many long narrow passages hardly wider than a handcart, there are few obvious signs of large scale planning or grids or axial planning. The city has an ad-hoc unity, only excepting the park in the East nowadays used for the Venice Biennale. It was built in

the geometrical French style with wide tree and statue-lined boulevards when the city lost its independence following Napoleon's conquest of 1805. It is conspicuously different in style to the rest of the city and was intended to be so as a symbol of a new order.

Figure 3:

Axial maps of Venice, canals and roads

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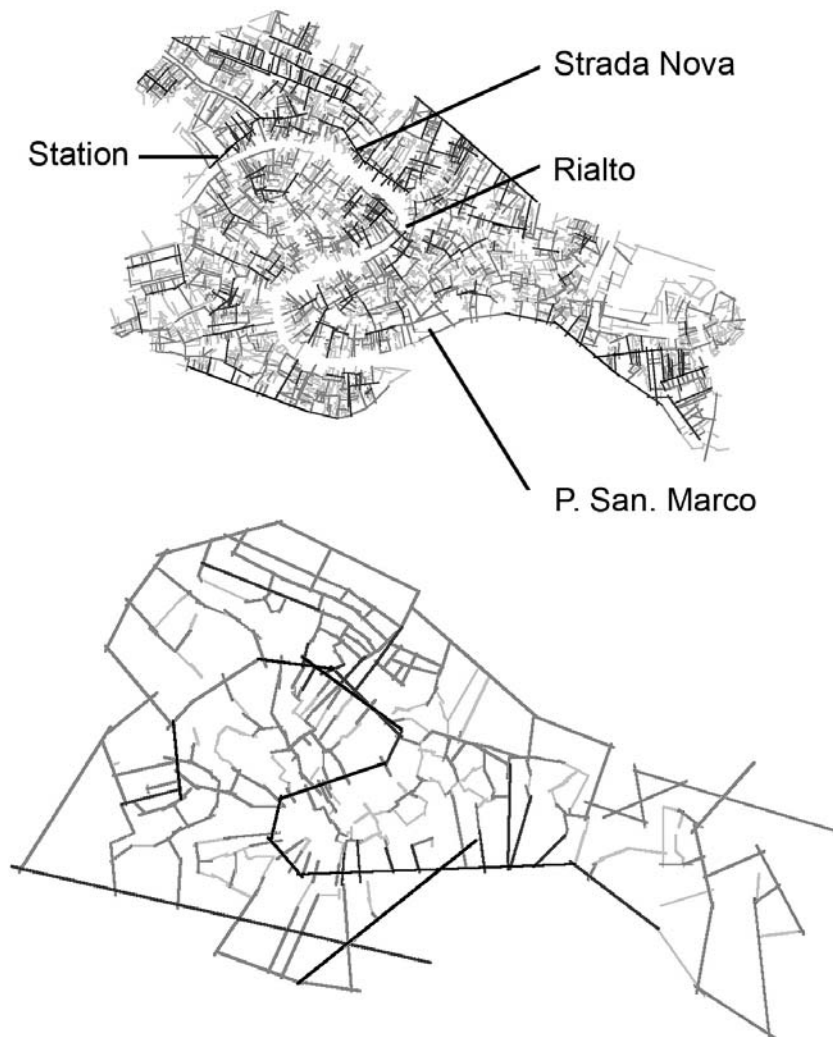


Axial Map Analysis

Axial maps for canals and streets, together and separately, are shown in figure (3). When the maps on land and on water are laid one over the other they intersect by crossing orthogonally in a rather beautiful way resembling the conformal maps found in mathematics. Integrations of the pedestrian axial map, (using Depthmap 5.09 from UCL set at $r = 3$), are shown in figure (4). It is pleasing to report that the vistas marked with the darkest lines were, at least in our experience, also the busiest just as the integration process is supposed to predict. At the centre of Venice the busy Rialto Bridge, a double route over the Grand Canal, is dark, as are the congested

routes leading to it. The heavily trafficked route from the Station running north of the Grand Canal is also emphasised. Part of this route, known as the Strada Nova, is one of only two or three thoroughfares in Venice that resemble ordinary streets with shops on either side. It is unusually broad being about 6 m wide and is difficult to avoid it if one walks from east to west. The other street of this type, the Via Garibaldi that leads to the eastern end of the island, is also dark. Interestingly both these street are filled in canals so the integration analysis confirms what planners in Venice presumably sensed; that these important routes were more valuable to pedestrians than boats. In general the map emphasises the routes along the edge of the lagoon, (where they exist, it being impossible to walk continuously round the perimeter of the city). An exception is the waterfront east of the Piazza San. Marco. It is not highlighted particularly strongly although it is probably the busiest place in Venice, however it is dark on the canal axial map, telling us that traffic here comes from the water. This area is a hub for water traffic from outlying islands and the mainland, something our land axial map cannot represent. The axial map of the canals is far simpler and also confirms one's intuition that the Grand Canal carries the most traffic followed in importance by the routes orbiting the city. These, of course are where the public waterbuses are to be found, in general only gondolas and private boats venture down the smaller canals. Generally the map agrees with a visitor's observations of Venice and confirms the usefulness of the integration process.

011-05

**Figure 4:***Integrated axial maps of Venice*

Scaling In The Axial Map

So much for the integration of our axial map: at the University of Manchester we have been interested in scaling and depth in the built environment and have conducted experiments on the capacity of the environment to contain activity at different scales (Crompton, 2005, Crompton & Brown, 2006). Therefore, we were naturally interested in Carvalho and Penn's paper of 2004 about scaling in axial maps. They discovered that lengths of lines in axial maps of some 36 cities were hyperbolically distributed and that the cities fell, broadly speaking, into two classes. We decided to test their result by applying their method to Venice.

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To do this Zipf plots were drawn for lines in the axial maps by ranking them in order of length, l , with the longest having rank $r = 1$, and then plotting $\log(l)$ against $\log(r)$. The results are shown in figure (5). In the graphs long axial lines are represented by points to the left and short lines by points to the right, the points dive away at the right because there are insufficient numbers of short lines to keep it going. What is significant about these Zipf plots is that they are linear over much of their range. In the shaded regions $\log(l)$ and $\log(r)$ are linearly related, in other words:

$$\log(l) = -(1/a) \log(r) + \text{const.} \quad [1]$$

(Length l , rank r , $-1/a$ is the gradient, so expressed to match the notation of Carvalho & Penn).

From this we obtain the scaling relationship ⁱⁱ:

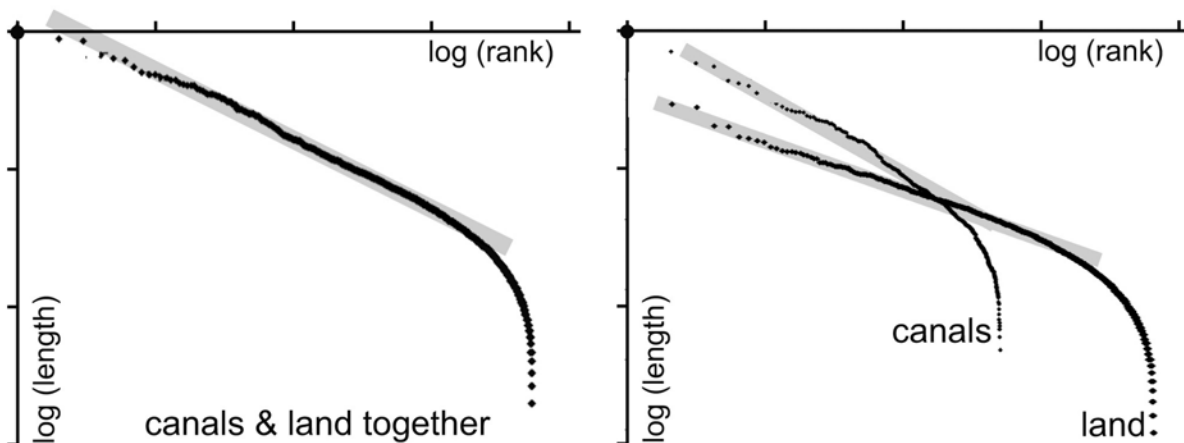
$$l = \text{const. } r^{-1/a} \quad [2]$$

$P(L > l)$ is the probability that a random variable L is bigger than l .

It is equal to $[r/\text{total number of lines}]$, so it follows that:

$$P(L > l) = \text{const. } l^{-a} \quad [3]$$

Figure 5:
Zipf plots of axial line lengths in Venice



This is the form of a hyperbolic relationship, (Mandelbrot, 1983). The value of a is determined by the distribution of line lengths and is an intensive rather than an extensive variable. Intensive qualities are perceptible locally, like density and temperature and its value does not depend upon the size of the city, so if only half the city were considered, perhaps by cutting it in two or by discarding half the axial lines at random then its value would be unchanged. It tells us something about space in the city that ought to be perceptible wherever we stand.

Carvalho and Penn measured the value of a for thirty-six cities and discovered that it generally fell into one of two bands. There were cities for which $a \sim 2$, (which they called blue) and those for which a

~ 3 , (which they called red). They hypothesised that cities with a ~ 2 possess open space alignments that cross the whole city, whereas cities with a ~ 3 do not. Red cities, ($a \sim 3$), included, Istanbul, London, Manchester, York, Bristol, Athens, Norwich, Nottingham and Hong Kong, and it does seem true that they lack axes that cross the whole city. Blue cities, ($a \sim 2$), included Bangkok, Tokyo, Milton Keynes, Pensacola, Seattle, Barcelona, Eindhoven, Shiraz and Kerman. Many of these cities do indeed have strong global grids and structures at the scale of the city. The lowest values of a were found for the most extreme cities of this type, namely Las Vegas and Chicago. When the gradient of the Zipf plot in figure (5) was measured it was found that for the 5274 lines in Venice, $a = 2.1$, placing Venice in the blue category as a city with large open space alignments. We need look no further than the Grand Canal to see such a city-sized open space, and to Jacopo de Barbari's aerial map to see it drawn for the first time as it might appear to a single glance.

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The large structures that distinguish cities of the $a \sim 2$ type are often transportation features such as big roads. Do these really belong to the city or could we see them as being laid over cities of the older type like the grids of urban motorways that the Buchanan Report (1963) proposed to be laid over English towns. When one looks at plans of cities such as Chicago one is inclined to say that its huge grid is actually deeply embedded in that city and a separation of global and local structures is not possible. Even for old cities that have had large scale roads built through them a separation into primary and secondary structures will be to an extent arbitrary, but it is certainly possible in Venice where all we have to do is to look at the water and land individually. When the axial maps for streets and canals were considered separately, as shown in the lower part of figure (5), both were seen to display scaling but with different values of a , namely 3.0 for streets and 1.72 for canals. This holds true over one and a half orders of magnitude for both canals and streets, [from 20 to 700 m for streets and from 100 to 2400 m for canals]. Probably this range could be increased if the map could be taken inside courtyards and gardens.

Venice may be therefore said to have a double fractal structure. Using Carvalho and Penn's model it can be thought of as a combination of two types of city and may be said to belong to both the blue and red type of city at the same time. Venice is formed by the intersection of two fractal circulation systems, an open one for commerce and a more closed one for everyday life, like the intersections of two worlds in one of Escher's bravura drawings. This is a very appropriate form for a city that is a byword for civilised living yet managed to handle most of the Levant trade for five hundred years. The first, explored on foot, is complex, intimate and lacks large structures. The second contains city-sized structures and is used for longer journeys, for commerce and for entering and leaving the city. The first is inward looking whereas the second is connected to the world, the Grand Canal being only one stage in a journey that connects water gates in dwellings to quaysides across the world. Perhaps our axial map ought to be extended to include channels in the lagoon, then the Adriatic, then sea routes throughout the Mediterranean

Hyperbolic Distributions

Hyperbolic distributions are beautiful and subtle. When the value of a is close to one they are called $1/f$ distributions and the axial map for the canals may be said to be of that type. They are often found in nature, reasons for their ubiquity and their significance may be found in Salinger & West, (1999). One of the oddities of a set of hyperbolically numbers is that their average value is a rather nebulous quantity and considerable care must be taken in calculating it.

Supposing we have a set of lengths distributed as in equation [3] with $a > 1$, then the average will only exist provided we set a lower limit to the lengths we will consider and if we lower that limit then the average decreases. If this lower limit is set at l_0 , then it is not difficult to show that the average value, the expectation value, is given by $a/(a-1) l_0$. The expectation values in Venice are $2.4 l_0$ for canals and $1.5 l_0$ for streets. These values give us some idea of how quickly the distribution grows from small to big. Suppose we set our minimum axial line for streets at 30 m long, a plausible size based on our survey, then the average will only be 45 metres long, even though the lines range up to a maximum of 700m. Forty-five metres represents a fair bet as to the length of a line chosen at random, it is, so to speak, what probably lies around the corner. The fact that it is only slightly longer than the minimum shows that nearly all the axes are short and that in Venice the detail dominates in much the same way that a component of a tree chosen at random is more likely to be twig than a branch. On water things are more spacious; the expectation value is 2.4 times the minimum value telling us that lengths are skewed further towards larger values compared with lines on the land.

Mandelbrot, (1983), has some droll paradoxes based on expectation values of hyperbolically distributions that have their counterpart in Venice. Suppose on a misty day you can only see for 100 metres. (i) With your back to a wall the route ahead vanishes into the fog; what is its probable length? On land the answer is an additional 50 m, but if you are on a canal, an additional 140 m: so in misty Venice the world is bigger in a boat than on foot. (ii) Walk 50 m along that street, if you still cannot see the end in the fog what is its probable length now? The answer is $150 \times 1.5 = 225$ m, that is, an additional 75 m beyond the fog: has the end of the street magically moved away as you walked?

These curiosities give an idea of how slippery hyperbolic distributions can be. Their average value is ill defined and they possess no middle value or any characteristic length. In fact hyperbolic distributions are scaling, meaning that they are scale free, so that they appear unchanged when uniformly expanded or shrunk, and except at their extremities may be laid over to match stretched copies of themselvesⁱⁱ. It is because they resemble resized copies of themselves that they are called fractal. This cannot happen with distributions in which a particular size predominates because in that case resizing can be detected when the characteristic size is seen to change. This brings us to an interesting question: if the distribution of axial line lengths in a city is hyperbolic could we tell if it was uniformly shrunk in size? If a city simply consisted of a series of spaces then the answer ought to be that we could not tell, but of course there are always things with fairly fixed sizes such as people, doors, windows and cars that would give the game away if they got smaller. Let them remain at their normal size and shrink the public space around them. Some sort of collision will occur when we are no longer able to fit cars down the roadsⁱⁱⁱ. Let us dispose of them and go on foot, how far could you go on making things smaller and still manage live in the city? The furthest you could go before things become difficult, we suggest, is about the scale of Venice where the narrowest paths are about two metres wide. What we are in effect doing is sliding down the hyperbolic distribution and living where the axial lines are smaller. And yet we still have the same scale-free mix of small medium and large spaces as before, the only difference being that we may have to go round a few more corners to get to the large spaces. In other words we can shrink the city and it will make little difference to how we read it provided we avoid large objects of known size, cars in particular.

Our prediction that we will be relatively insensitive to changes in scale if the sizes of spaces are distributed hyperbolically is something that

could be tested. Some support for this idea came from an experiment we performed in 2004 to see if small-scale places with no cars were felt to be larger than normal cities with traffic. One of our experiments compared distance perception in Portmeirion, a picturesque Italianate village in North Wales with Manchester. Our subjects felt that distances in the car-free picturesque village were about 2.5 times longer than equal length walks along a busy road with traffic in the city, (Crompton & Brown, 2006). This brings us to a question that has interesting implications for the efficient use of space.

How Big is Venice, Really?

Venice is actually rather small. Although it does not always seem to be so as the following three examples show:

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(i) The Venetian axial map contained 5274 lines. Using figures from Carvalho and Penn's survey this may be compared with; Barcelona - 5575 lines; New Orleans - 4846 lines; and Manchester - 4308 lines. An axial map is a sum of vistas and its size may be treated as a measure of spatial complexity so we can say that Venice is similar to these cities. Now compare Venice to Manchester; both cities have their value of a close to 3, but whereas the average length of an axial line in Venice is 46 m the average in Manchester is 130 m. Venice has 12.0 lines/ha, Manchester has 2.51 lines/ha^{iv}. So although Manchester and Venice are similar in spatial complexity Manchester covers four to five times the area of Venice. And Manchester is by no means a spacious city, it was densely developed in the nineteenth century and the only open spaces that remained are hardly more than widened streets rather like the Campi in Venice.

(ii) In Venice one often finds oneself calculating whether it will be quicker to find and wait for a waterbus or to plunge into the passageways. Usually it is quickest on foot. Of course one does the same thing in London in calculating whether it is better to take the underground or walk, but only for the very shortest journeys. In Venice the whole city is accessible for a brisk walker with a map. To travel from the Station in the East to the Via Garibaldi in the far West along the shortest route is to walk about 3100 m; you can walk right across Venice about half an hour if you don't get lost, something that cannot be done in Barcelona, Manchester or New Orleans.

(iii) In figure (6) the parts of Venice visited by tourists are shown darkened next to Central Park at the same scale. The dark areas have very nearly the same area, namely 3.33 ha. This seems surprising, somehow one feels that Venice, with its art, history and wealth ought to be bigger than a public park, but it is not.

Venice, we hypothesise manages to seem so large because its public spaces have a scale free hyperbolic distribution of sizes as shown by the axial map. Although it was built at the scale of a pedestrian one is generally unaware of its smallness because not everything in it is small; it has medium and large spaces, in fact it has the same spatial complexity as a city with four times its area. It simply occupies a range of sizes at the smaller end of the same hyperbolic distribution. We perceive it as picturesque rather than squashed and it comes as a surprise when its true size is exposed when eight-deck cruise-liners enter the channel between the Guidecca and the city and tower above its buildings and churches. Venice was able to be so dense and have such a small scale because it separated its transport system into two interlocking networks with different scales. The problem of transport in cities and the scale of vehicles it employs is probably an important limit on how compact a city can become.

There is plenty of evidence that we are predisposed to respond favourably to fractal environments and that they are in a deep sense

normal and natural, in fact our nervous system may have evolved to expect our environment to be fractal, (see Yang & Purves, 2003, for a good example of this). In fractal environments there is spatial variety, and if that variety is spoilt with large numbers of object of the same size, be they cars or identical buildings then we are doing something unnatural and possibly oppressive. If we want to fit more into our world we need to do something more sophisticated than just making everything smaller. The pleasures of Venice have much to teach us about the importance of variety and spatial depth in our cities. Venice shows us that we can build at a high density if sizes of spaces are hyperbolically distributed. This incorporates large and small in a natural way. It is the key to building more in a given space and a clue to a new approach to sustainable development that will use land more economically.

011-10

Figure 6:

Venice and Central Park to the same scale



Accomplishment; We thank Marisa Scarso of CIRCE at the Università di Venezia (i.e. the Cartographic Institute at the University of Venice) for providing the AutoCAD model of Venice, and Ahmed Mohamed Refaat Mostafa of Manchester University for his work analysing the axial map.

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- i. Two versions of the map for canals were made since it was not clear how to handle the axial lines along canals that left the city and headed into open water. Those on the south ended on the Giudecca, but a few on the North headed out into open water. In the first version the routes of boats following a path parallel and close to the shore were treated as axes and axial lines going out to sea were stopped where they crossed them. In the second case lines were drawn out into open space about as far as Murano, which is about as far as one could see. The differences between the two are that the first had $a = 1.72$ and the second had $a = 1.61$, the lower value reflecting the presence of longer lines. The differences between these values gives an indication of their accuracy, and gives weight to the observation that they are significantly less than the value for land axes.
 - ii. Equation [2] is scaling because, letting b be a constant, if $P(L>l) = l^{-a}$ then $P(L>b) = b^{-a} \cdot l^{-a} = b^{-a} P(L>l)$, i.e. the same as the original only multiplied by a factor b^{-a} .
 - iii. Of course cars could be made smaller, Crompton, 2005 describes what happens when this is done; the number of cars that can be accommodated goes up hugely.
 - iv. For comparison, Carvalho and Penn found 15969 lines in London, 73753 in Tokyo, 1773 in York. Our own axial map of part of Manchester's centre contained 850 lines in an area of 3.38 sq. km. As in Venice we drew the axial map up to the public - private boundary.

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